Phys 218 — Challenge Exam Formulae

Trigonometry and Vectors:

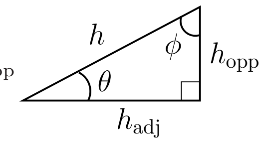
$\sin 30^\circ = \cos 60^\circ = \frac{1}{2}$ $\quad$ $\sin 36.9^\circ \approx \cos 53.1^\circ \approx \frac{3}{5}$

$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$ $\quad$ $\sin 53.1^\circ \approx \cos 36.9^\circ \approx \frac{4}{5}$

$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$

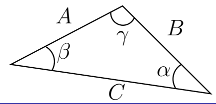
$h_{\rm adj} = h\cos\theta = h\sin\phi$ $\quad$ $h^2 = h_{\rm adj}^2 + h_{\rm opp}^2$

$h_{\rm opp} = h\sin\theta = h\cos\phi$ $\quad$ $\tan\theta = \frac{h_{\rm opp}}{h_{\rm adj}}$



Law of cosines: $C^2 = A^2 + B^2 - 2AB\cos\gamma$

Law of sines: $\dfrac{\sin\alpha}{A} = \dfrac{\sin\beta}{B} = \dfrac{\sin\gamma}{C}$



$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ $\quad$ $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z = AB\cos\theta = A_\parallel B = AB_\parallel$

$\vec{A}\times\vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$

$|\vec{A}\times\vec{B}| = AB\sin\theta = A_\perp B = AB_\perp$ (direction via right-hand rule)

Kinematics:

translational	rotational
$\langle\vec{v}\rangle = \frac{\vec{r}_2-\vec{r}_1}{t_2-t_1}$ $\quad$ $\vec{v} = \frac{d\vec{r}}{dt}$	$\langle\omega\rangle = \frac{\theta_2-\theta_1}{t_2-t_1}$ $\quad$ $\omega = \frac{d\theta}{dt}$
$\langle\vec{a}\rangle = \frac{\vec{v}_2-\vec{v}_1}{t_2-t_1}$ $\quad$ $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$	$\langle\alpha\rangle = \frac{\omega_2-\omega_1}{t_2-t_1}$ $\quad$ $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$
$\vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t')\,dt'$	$\theta(t) = \theta_0 + \int_0^t \omega(t')\,dt'$
$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t')\,dt'$	$\omega(t) = \omega_0 + \int_0^t \alpha(t')\,dt'$

— constant (linear/angular) acceleration only —

translational	rotational
$\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$\vec{v}(t) = \vec{v}_0 + \vec{a}t$	$\omega(t) = \omega_0 + \alpha t$
$v_x^2 = v_{x,0}^2 + 2a_x(x - x_0)$ (and similarly for y and z)	$\omega_f^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$\vec{r}(t) = \vec{r}_0 + \frac{1}{2}(\vec{v}_i + \vec{v}_f)t$	$\theta(t) = \theta_0 + \frac{1}{2}(\omega_i + \omega_f)t$

Energy and Momenta:

translational	rotational
	$\vec{\tau} = \vec{r}\times\vec{F}$ and $|\vec{\tau}| = F_\perp r = Fl$
$K = \frac{1}{2}Mv^2$	$K_{\rm rot} = \frac{1}{2}I_{\rm tot}\omega^2$
$W = \int \vec{F}\cdot d\vec{r} \xrightarrow[\text{force}]{\text{const}} \vec{F}\cdot\Delta\vec{r}$	$W = \int \tau\, d\theta \xrightarrow[\text{torque}]{\text{const}} \tau\,\Delta\theta$
$P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$	$P = \frac{dW}{dt} = \vec{\tau}\cdot\vec{\omega}$
	$\vec{L} = \sum \vec{r}\times\vec{p}$
$\vec{p}_{\rm cm} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots$	$= I_1\vec{\omega}_1 + I_2\vec{\omega}_2 + \ldots$
$= M\vec{v}_{\rm cm}$	$= I_{\rm tot}\vec{\omega}$
$\vec{J} = \int \vec{F}dt = \Delta\vec{p}$	
$\sum \vec{F}_{\rm ext} = M\vec{a}_{\rm cm} = \dfrac{d\vec{p}_{\rm cm}}{dt}$	$\sum \vec{\tau}_{\rm ext} = I_{\rm tot}\vec{\alpha} = \dfrac{d\vec{L}}{dt}$
$\sum \vec{F}_{\rm int} = 0$	$\sum \vec{\tau}_{\rm int} = 0$
if $\sum F_{{\rm ext},x} = 0,\ p_{{\rm cm},x} = \text{const}$	if $\sum \tau_{{\rm ext},z} = 0,\ L_z = \text{const}$

— Work-energy and potential energy —

$W = \Delta K$ $\qquad$ $E_{{\rm tot},i} + W_{\rm other} = E_{{\rm tot},f}$

$U = -\int \vec{F}\cdot d\vec{r}\ ;\quad U_{\rm grav} = Mgy_{\rm cm}\ ;\quad U_{\rm elas} = \frac{1}{2}k\Delta x^2$

$F_x(x) = -dU(x)/dx \qquad \vec{F} = -\vec{\nabla}U = -\left[\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right]$

Quadratic:

$$ax^2 + bx + c = 0 \quad\Rightarrow\quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Derivatives:

$\frac{d}{dt}(at^n) = nat^{n-1}$

$\frac{d}{dt}\sin at = a\cos at$

$\frac{d}{dt}\cos at = -a\sin at$

Integrals:

if $f(t) = at^n$, then $\begin{cases} \int_{t_1}^{t_2} f(t)dt = \frac{a}{n+1}(t_2^{n+1} - t_1^{n+1}) \\ \int f(t)dt = \frac{a}{n+1}t^{n+1} + C \end{cases}$

$(n \neq -1)$

$\int \sin at\, dt = \frac{-1}{a}\cos at$

$\int \cos at\, dt = \frac{1}{a}\sin at$

Constants/Conversions:

$g = 9.80\ {\rm m/s^2} = 32.15\ {\rm ft/s^2}$ (Earth, sea level)

$\approx 10\ {\rm m/s^2} \approx 33\ {\rm ft/s^2}$

$G = 6.674\times10^{-11}\ {\rm N\cdot m^2/kg^2}$ $\qquad$ $1\ {\rm mi} = 1609\ {\rm m}$

$1\ {\rm lb} = 4.448\ {\rm N}$ $\qquad$ $1\ {\rm ft} = 12\ {\rm in}$

$\Leftrightarrow 0.454\ {\rm kg}$ (Earth, sea level) $\qquad$ $1\ {\rm in} = 2.54\ {\rm cm}$

$1\ {\rm rev} = 360^\circ = 2\pi$ radians

Circular motion: $\quad a_{\rm rad} = \dfrac{v^2}{R} \qquad a_{\rm tan} = \dfrac{d|\vec{v}|}{dt} = R\alpha$

$T = \dfrac{2\pi R}{v} \qquad s = R\theta \qquad v_{\rm tan} = R\omega$

Relative velocity: $\quad \vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}$

$\vec{v}_{A/B} = -\vec{v}_{B/A}$

Forces:

Newton's: $\sum \vec{F} = m\vec{a},\quad \vec{F}_{B\text{ on }A} = -\vec{F}_{A\text{ on }B}$

Hooke's: $F_x = -k\Delta x$

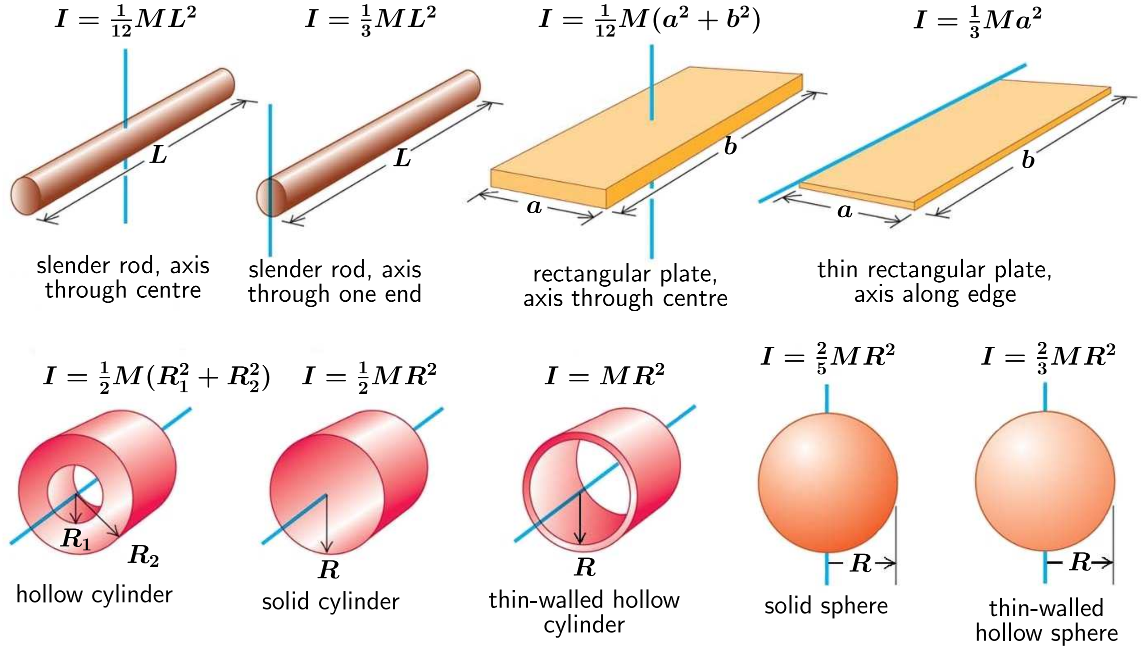
friction: $|\vec{f}_s| \le \mu_s|\vec{n}|,\quad |\vec{f}_k| = \mu_k|\vec{n}|$

Centre-of-mass:

$$\vec{r}_{\rm cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots + m_n\vec{r}_n}{m_1 + m_2 + \ldots + m_n}$$

(and similarly for $\vec{v}$ and $\vec{a}$)

Moments of inertia:



⇝ For a point-like particle of mass M a distance R from the axis of rotation: $I = MR^2$

⇝ Parallel axis theorem: $I_p = I_{\text{cm}} + Md^2$